

## **A Non-Fickian Mixing Model for Stratified Turbulent Flows**

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### **LONG-TERM GOALS**

The long term goal of this project is to develop a better understanding of oceanic processes in the range of 100 m to 10 km, in the so-called submesoscale range. In particular, it is important to explore and find out whether and what type of submesoscale instabilities exist, how they are connected to both larger scale and smaller scale motions, and to what extent they influence transport processes in the ocean. Another important objective of this project is to test how well subgrid-scale (SGS) models for large eddy simulations (LES) work in the presence of backward energy cascade that may be characteristic in submesoscale motions. Another long term objective of this effort would be to improve the predictive skill of the Navy numerical models for submesoscale transport in the ocean.

### **OBJECTIVES**

My main objective has been to model upper ocean mixed layer instabilities, investigate their behavior and try to develop sampling strategies using synthetic drifters and tracers prior to the first LatMix cruise. In addition, I have put in a significant effort during the first LatMix cruise that took place in June 2011.

### **APPROACH**

The work is based on large eddy simulations using the non-hydrostatic spectral element model Nek5000. Also, software has been developed in order to automatically download and make available MODIS raw SST data over the selected region in the first LatMix cruise.

### **WORK COMPLETED**

- 1) A paper containing extensive testing of passive particle and tracer sampling of the mixed layer instability fields generated by LES has been published (Özgökmen et al., 2011a).
- 2) A paper in which I consider some quite challenging realistic constraints on tracer and particle release in mixed layer instability fields has been completed and submitted for publication (Özgökmen et al., 2011b).

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- 3) I have facilitated the incorporation of 20 NOAA-AOML drifters in the LatMix cruise and participated (remotely, from land) extensively in the discussions during the cruise, in particular regarding the release of NOAA drifters.

## RESULTS

### *1) Sampling of mixed-layer instability using passive tracers and particles:*

Our main goal here is to create fields of mixed layer instability using LES, and then examine the efficiency of passive tracers and drifters in sampling these submesoscale flows. This study is a continuation of Özgökmen et al. (2011a), but differs in that we consider sampling strategies subject to the following experimental constraints:

- (a) The flows of interest take place inside mixed layers that are about 25 m deep, when compared to the lateral size of interest of 10 km. Modeling of flows with such high geometric aspect ratio of 1/400 demands schemes with low numerical dissipation and the use of high Peclet numbers in order to avoid an unrealistic conversion of available potential energy driving the system to background potential energy by spurious mixing, instead of kinetic energy via flow instabilities.
- (b) The tracer patches are likely to be approximately 500 m long, 50 m wide and 2 m thick (all dimensions are estimates within a factor of about two). The tracer is to be injected at the mixed layer base. This is mainly to reduce the effects of the diurnal cycle and wind-driven flows near the surface on tracer evolution.
- (c) Due to the decay rate of fluorescein needed for airborne detection of the dye by laser, LIDAR observation period is limited to 36 hours, while other dye sampling can last up to 120 hours.
- (d) About 18 drifters are available with a temporal sampling of 30 minutes for a total period of about 30 days.

These constraints are by no means the only ones that are encountered in the field. Here, we will assume that there are no spatial or temporal errors in observing the dye or drifters, and the velocity field is frozen during the period of launching the observational assets.

**Description of the flow field:** From the initial perturbation consisting of various wavelengths, modes consistent with the scale of the fastest growing mode propagate towards the higher density perturbation region (northern side) of the front during the first week of the instability (Fig. 1a,b). The circulation inside these modes is anticyclonic (clockwise or negative relative vorticity,  $\zeta$ ). Under the assumption that tilting/twisting and solenoidal terms in the vorticity budget are negligible, the potential vorticity (PV) can be expressed as  $q=(f+\zeta)/h$ , so that the initial PV in the mixed layer is  $q_1=f/h_0$ . During the rotating gravitational adjustment, the water parcels in the mixed layer are advected such that their height  $h_1$  reduces. Under the PV conservation  $q_1=f/h_0=(f+\zeta_1)/h_1$ . Since  $h_1<h_0$ ,  $\zeta_1<0$ , or anticyclonic circulation must develop inside these features. The zones in between these modes are subject to shear of reverse direction (cyclonic), which is also supported by PV conservation arguments for the deeper layer;  $q_2=f/(H-h_0)=(f+\zeta_2)/h_2$ . Since  $h_2<(H-h_0)$ , one gets  $\zeta_2>0$ . We note that the shear layers between the fastest growing surface modes continue to release small cyclonic vortices, that populate the mixed layer base (Fig. 1b). There are occasional indications of Kelvin-Helmholtz type shear instability between the surface modes (Fig. 1c). The weaker yet persistent geostrophic current caused by the front

(from left to right of the domain, or with the deeper, high pressure perturbation pool on its right) appears to exert control and significantly influence the flow evolution. First, it tends to disrupt the fastest growing modes, causing detachment of anticyclonic surface eddies. At longer times, anticyclonic surface eddies spin down, and the cyclonic shear along the northern side of the front creates a large number (about 10) of cyclonic spiraling eddies near the surface. Spiral eddies are commonly observed in the ocean, and their formation in this simulations appears to be consistent with the mechanism outlined in Fig. 34 of Munk et al. (2000).

***Sampling with realistic-size dye strips:*** Realistic size tracer strips are used to sample the mixed layer base. One of the striking first impressions one gets is how small the dye patches are with respect to the submesoscale features in the flow field (Fig. 2). Subsequently, multiple patches need to be considered, and positioning of the patches could play an important role given the limited observation period. It is assumed that the topology of the mixed layer base can be mapped to some degree of accuracy using ship board and/or profiling instruments. Two types of features are selected. The first type of features are centers of eddies that populate the mixed layer base (Fig. 2a). These are essentially elliptic stagnation points that tend to trap particles. Second type of features are regions in between the eddies that are hyperbolic due to strain exerted by the eddies (Fig. 2b). In order to accelerate the formation of tracer patterns, 10 patches of tracers are launched in a cross configuration (5 independent locations) at the mixed layer base at 25 m. The launch time is chosen to be  $t=17.3$  days (Fig. 1c), at which point the system exhibits well-developed instabilities, while allowing adequate time for integration before the closed boundaries influence the turbulent features.

Figs. 2c,d indicate that rotation and trapping associated with the elliptic eddies, and stretching along the outflowing manifolds of hyperbolic regions in between the eddies are apparent in the tracer fields sampled after 36 hours. This result shows the feasibility of identifying some of the pronounced features in the mixed layer base using well-resolved sampling of dye, despite the translation of features by several km over the observation period.

Nevertheless, one of the primary obstacles associated with dye measurements is that the concentration fields is not easy to convert into useful diagnostics of transport. For instance, while the dye behavior in Fig. 2 is qualitatively indicative of elliptic and hyperbolic regimes in the underlying flow field, we are not aware of any quantitative methods of how to convert  $C$  into eigenvalues of the velocity gradient tensor, or compute components of strain tensor. This is mainly due to two reasons. First, dye parcels cannot be tagged (apart from using drifters), and second, formation of gradients in the dye field by the action of coherent features starts to become compensated by diffusion at some point, most notably along the exponentially stretching (hyperbolic) branches of the flow field.

***Sampling with passive particles:*** The main disadvantage of drifters with respect to tracers is that they provide sparse information, as it is not feasible to release hundreds and thousands of them in ocean experiments, given present technology. Nevertheless, particles offer two advantages over tracers. They are not subject to diffusion, and they provide position information, from which it is quite easy to compute quantities relevant to transport. A diagnostic of interest is the scale-dependent Lyapunov exponent (FSLE, Artale et al, 1997), which is directly connected to turbulent and scalar fluctuations in the underlying flow field.

In order to obtain an accurate evaluation of the FSLE in this flow simulation, the field is sampled over an area of 2 km by 5.5 km using 3528 particles launched as triplets at 25 m depth (Fig. 3a). The particles composing the triplets are released 20 m apart, and the triplets are 100 m apart. Particles are

advected online for about 30 days with the model time step of 150 seconds, while the positions are recorded with 30 minute intervals, as per observational requirements.

The resulting FSLE plot (Fig. 4) identifies a constant-value plateau for the scale separation range from 20 m to 600 m and a fall off to Richardson regime for larger separation distances. The fall-off scale is related to the average eddy diameter in the sampled field, while the value of the maximum FSLE is the average stretching that the particles experience (Poje et al., 2010). The maximum value seems to be about  $0.4 \text{ days}^{-1}$ .

Then, the following question is posed:

*How do we target a much smaller number of particles (18) to approximate this FSLE curve with the least error?*

It is helpful to use a quantitative criterion to select the target areas. Here we employ the Okubo-Weiss criterion  $Q$  (Okubo, 1970), which is a useful Eulerian metric to differentiate the hyperbolic ( $Q>0$ ) and elliptic ( $Q<0$ ) parts of a two-dimensional flow field. It is assumed that data from other in-situ instruments can help construct an approximate Okubo-Weiss map to help guide targeted drifter launches. A set of 5 targeted releases are carried out. The first release contains 6 triplets at strain-dominated regions ( $Q>0$ , between eddies), the second in elliptic regions ( $Q<0$ , inside eddies), and the third in  $Q\sim 0$  regions. The fourth and fifth launches contain two realizations of random combinations consisting of two of each type of release (so-called mixed releases).

Fig. 4 shows that the first launch starts with high FSLE (as expected) but drops almost to the true curve at separation scale of  $\delta \approx 100 \text{ m}$ , which seems to be the scale that the particles remain inside high strain initial regions. The launches in region two show low FSLE up to  $\delta \approx 600 \text{ m}$  (the average size of the eddies, as expected). The mixed launches are possibly the best performing. Nevertheless, perhaps the main message is that almost all of the releases are leading to quite accurate results, because there are many eddies and hyperbolicity in the mixed layer base, and given enough time, 18 particles seem to sample the dispersive statistics of these features quite satisfactorily.

**Remarks:** We conclude that the results presented here are supportive of the efficiency of Lagrangian observation platforms, and in particular, Lagrangian drifters in sampling rapidly-evolving submesoscale mixed layer flows. Future availability of cheaper, smaller and environmentally degradable drifters may permit their deployment in large numbers [on the  $O(1000)$ ] for better observing capabilities. In particular, floats with truly 3D position sampling capabilities, such as those developed by D'Asaro et al. (2003), are perhaps best suitable for complex near-surface flows.

## **2) Preliminary Results From the LatMix-2011 Cruise:**

The scale-dependent FSLE is computed on the basis of 20 NOAA SVP drifters (drogued at 15 m) released during the LatMix-2011 cruise in two patches; four were deployed at the beginning of the cruise, in the vicinity of low-strain Site 1, and the rest were deployed after two weeks when Site 1 become more active dynamically. The drifters have been deployed by pairs and triplets, which is suitable to compute relative dispersion.

Fig. 5 (left panel) shows a very interesting relative dispersion curve. At separation scales less than 100 km (below the mesoscale radius of deformation), we see a saturation of the FSLE at around  $0.5 \text{ days}^{-1}$ . This is a typical strain rate associated with mesoscale eddies ( $\Delta U \sim 0.5 \text{ cm/s}$  between the eddies of scale  $R \sim 50 \text{ km}$ , so  $\Delta U/R \sim 0.5 \text{ days}^{-1}$ ). This plateau lasts for scales between 100 km to 20 km. In the submesoscale range (separation scales smaller than 20 km and sampling error scale of 1 km), we observe a gradual and substantial increase in the relative dispersion up to  $15 \text{ days}^{-1}$  presumably by submesoscale motions.

Nevertheless, it appears that the ARGO positioning of the NOAA drifters contains fairly large errors (up to 1 km) and this issue is presently under investigation.

## IMPACT/APPLICATIONS

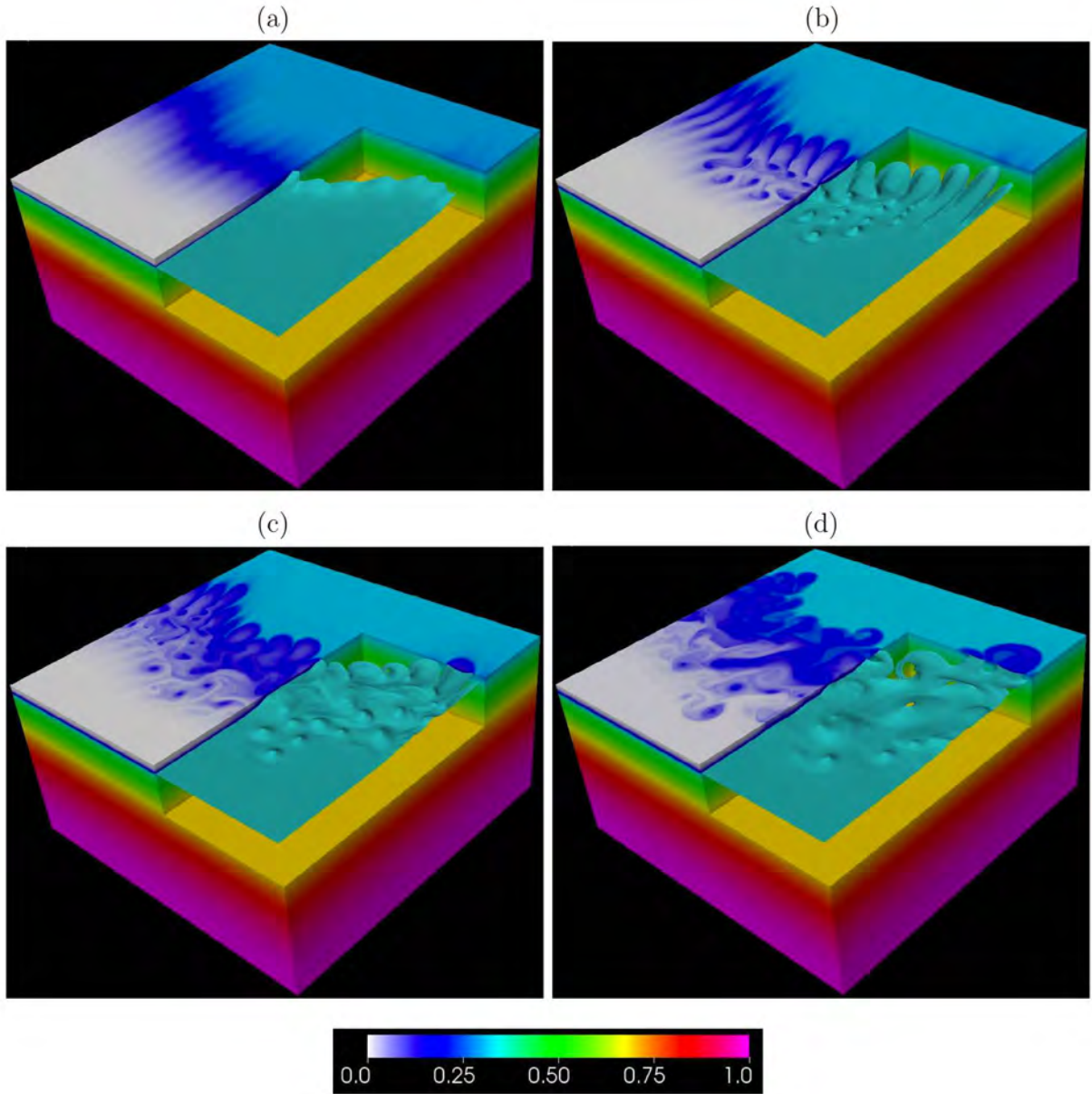
The scales considered in this project represent the range of scale of navy operations and thus anomalous currents and perturbations in the acoustic and optical environment that can affect a variety of navy operations. Understanding the motion in this range of scales is therefore critical to help improve the predictive capability of the existing Navy models.

## REFERENCES

- Artale, V., G. Boffetta, A. Celani, M. Cencini, and A. Vulpiani, 1997: Dispersion of passive tracers in closed basins: Beyond the diffusion coefficient. *Phys. Fluids*, 9:3162–3171.
- D’Asaro, E.A., 2003: Performance of autonomous Lagrangian floats. *J. Atm. Oceanic Tech.*, 20:896–911.
- Munk, W., L. Armi, K. Fischer and F. Zachariasen, 2000: Spirals on the sea. *Proc. Roy. Soc. London A*, 546, 1217-1280.
- Okubo, A., 1970: Horizontal dispersion of floatable particles in vicinity of velocity singularities such as convergences. *Deep-Sea Res.*, 17(3):445.
- Poje, A.C., A.C. Haza, T. M. Özgökmen, M. Magaldi and Z.D. Garraffo, 2010: Resolution dependent relative dispersion statistics in a hierarchy of ocean models. *Ocean Modelling*, 31, 36-50.

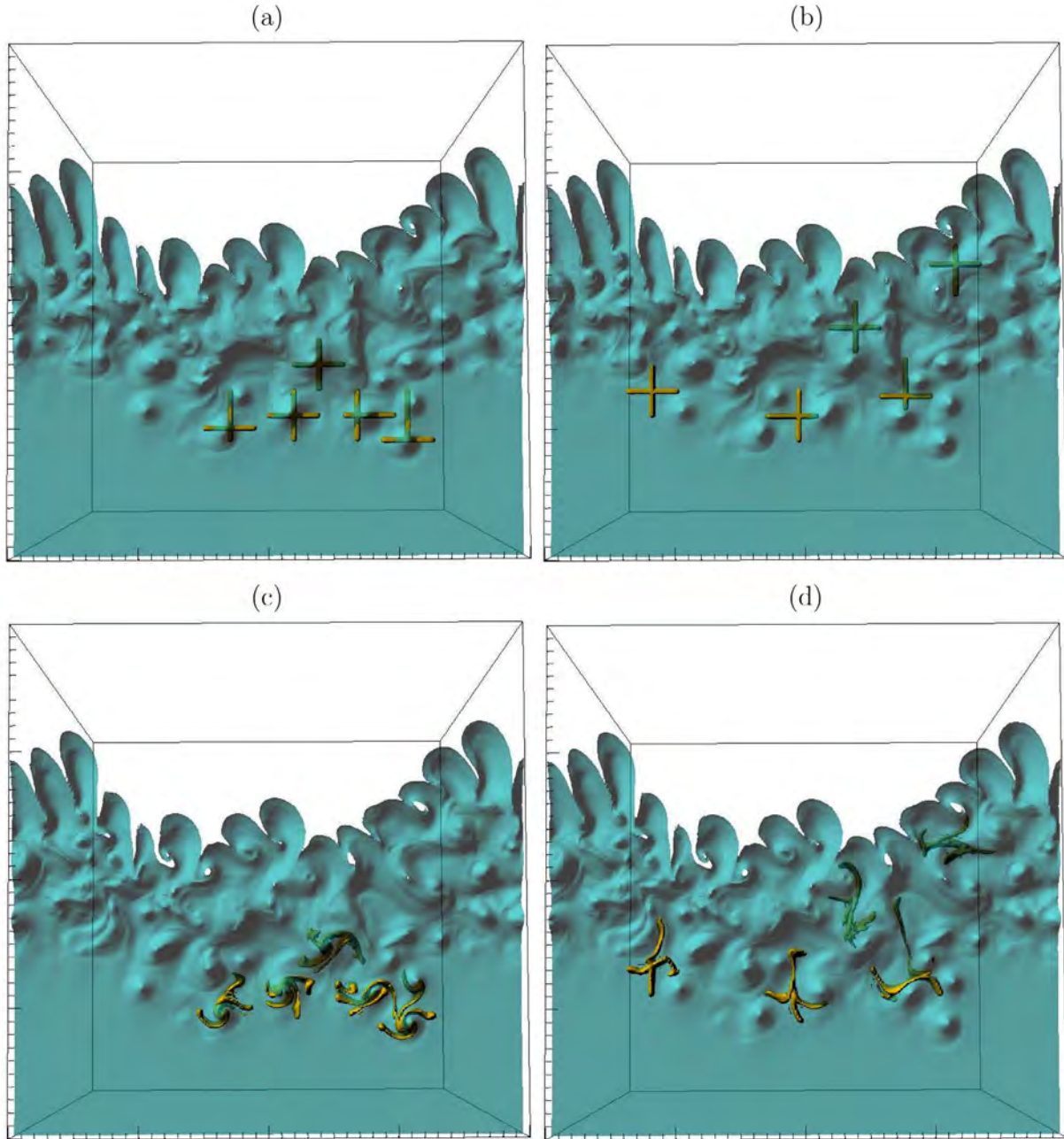
## PUBLICATIONS

- Özgökmen, T.M, P.F. Fischer, A.C. Poje and A.C. Haza, 2011a: Large eddy simulations of mixed layer instabilities and sampling strategies. *Ocean Modelling*, 39, 311-331 [published, refereed].
- Özgökmen, T.M. and P.F. Fischer, 2011b: CFD application to oceanic mixed layer sampling with Lagrangian platforms. *Int. J. Comp. Fluid Dyn.* [submitted, refereed].



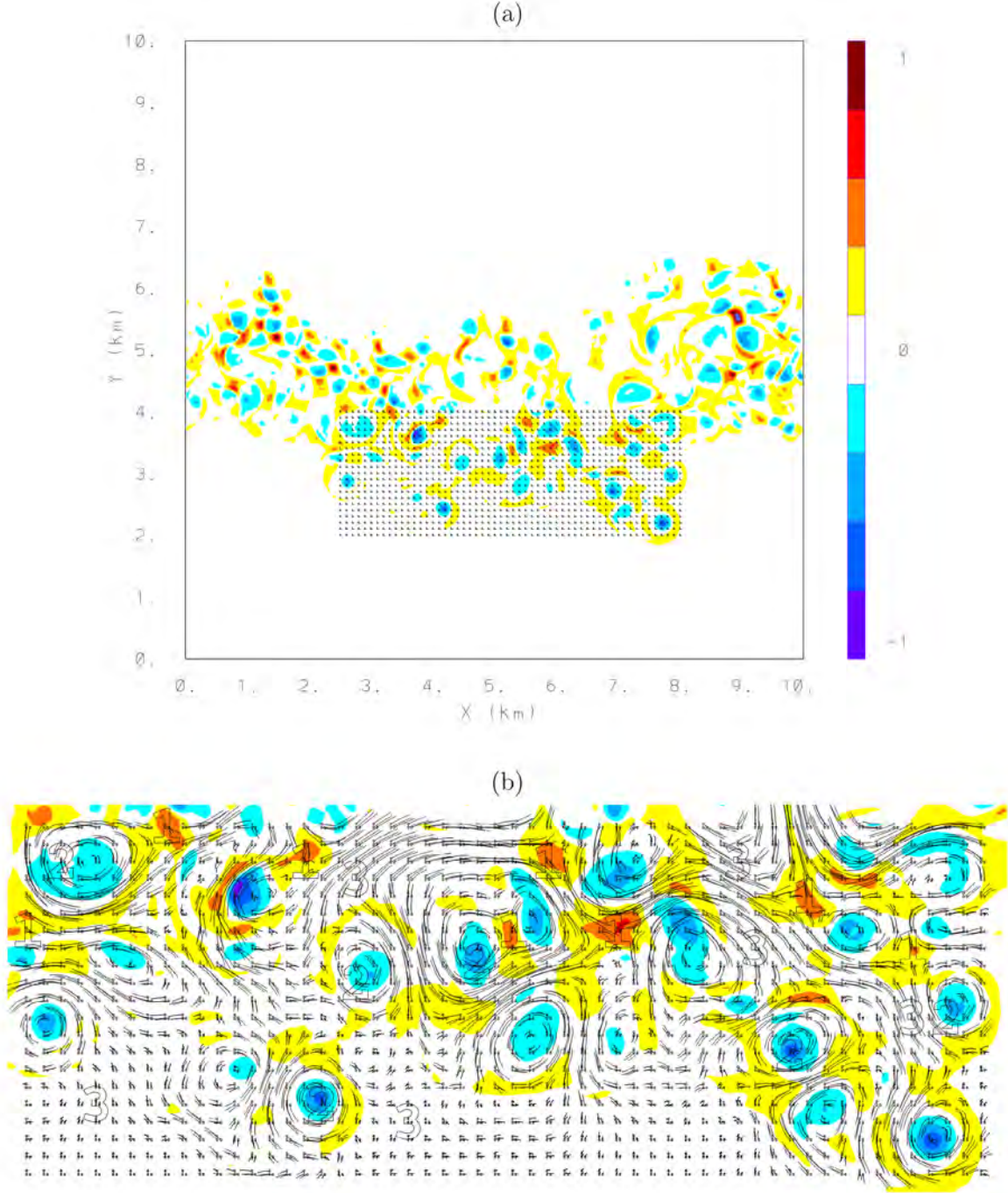
**Fig. 1:** *Density perturbation field from LES of mixed layer instability at (a)  $t=0$ , (b)  $t=11$  days, (c)  $t=17.3$  days and (d)  $t=29.5$  days. The surface plots show the isopycnal marking the base of the mixed layer. The vertical axis is stretched 10 fold with respect to the realistic perspective. The animation is available from: <http://www.rsmas.miami.edu/personal/tamay/3D/mli116-tr.mov>*



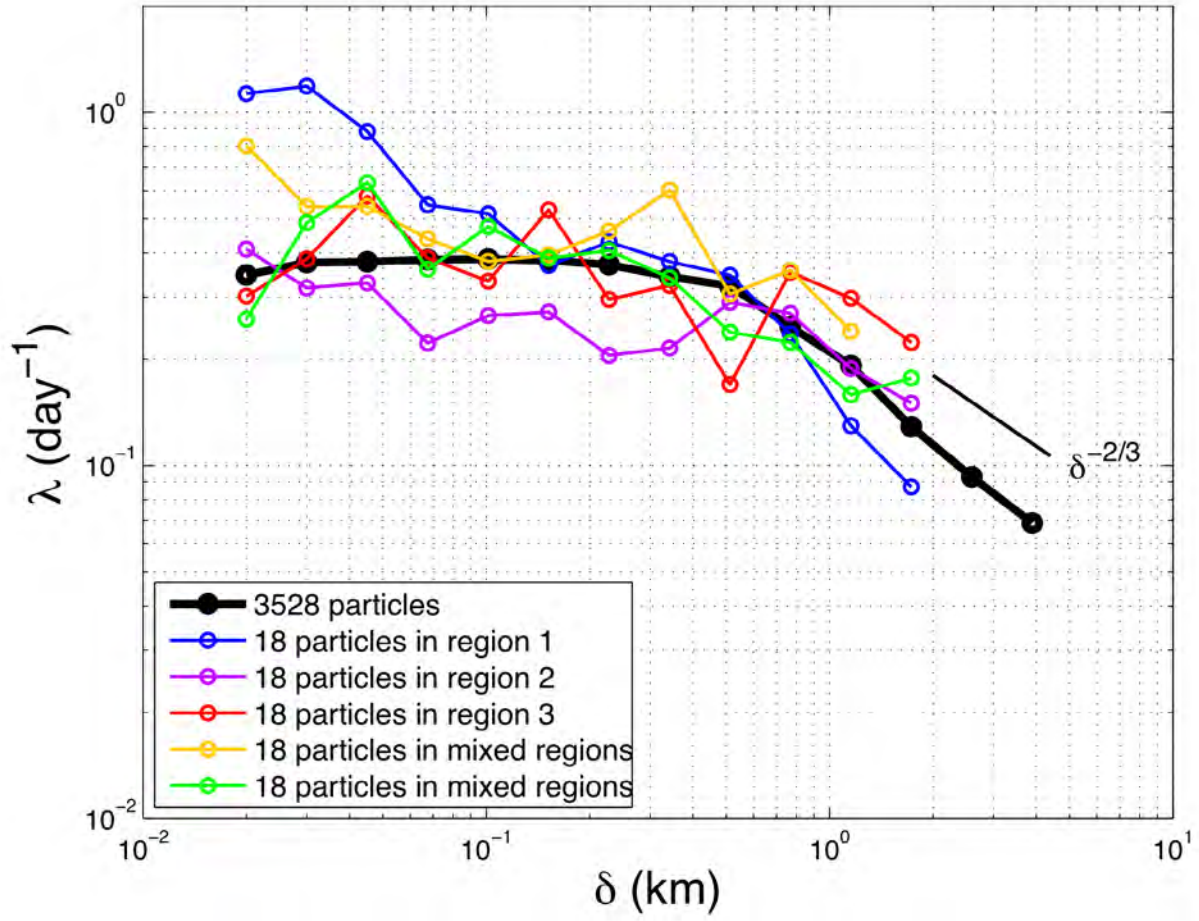


**Fig. 2:** Top view of the domain showing the initial locations of realistic size tracer strips in cross configuration positioned to sample (a) eddies in the mixed layer base, and (b) regions in between the eddies. The surface plot shows the shape of the mixed layer base at  $t=17.3$  days. The state of the tracer strips after 36 hours are depicted in (c) and (d). The animations are available from:  
<http://www.rsmas.miami.edu/personal/tamay/3D/mli121-tr-v1.mov> and  
<http://www.rsmas.miami.edu/personal/tamay/3D/mli122-tr-v1.mov>

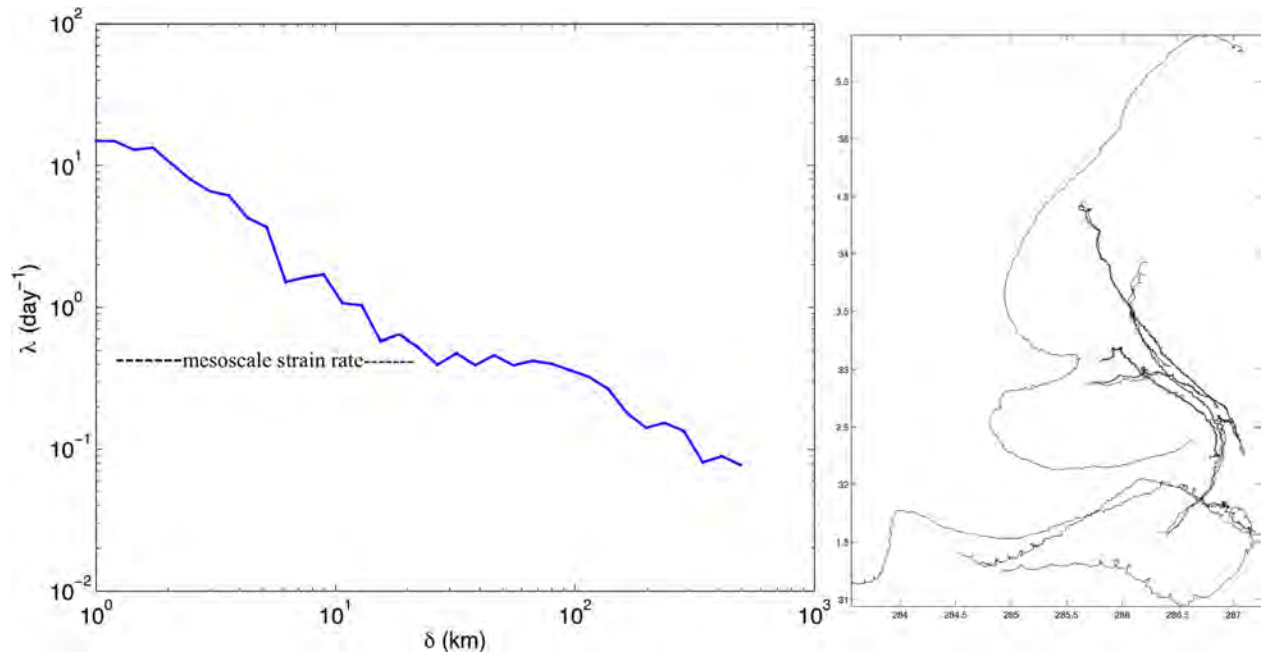




**Fig. 3:** (a) Okubo-Weiss parameter, normalized by mid-latitude Coriolis parameter,  $Q/f^2$ , at 25 m depth and  $t=17.3$  days, as well as the initial positions of 3528 particles launched in triplets. (b) Targeted launches of triplets in 6 strain dominated regions ( $Q>0$ , marked with '1'), in 6 vorticity dominated regions ( $Q<0$ , marked with '2') and 6 regions of low  $Q$  (marked with '3').



*Fig. 4: The scale-dependent FSLE from the full set with 3528 particles, and targeted sets each with 18 particles. The slope in the background indicates the Richardson dispersion regime.*



**Fig. 5: (left panel) Scale-dependent FSLE computed from 20 NOAA SVP drifters (right panel) launched during the 2011 LatMix cruise in the Gulf Stream region.**